Shear Correction Factors for Laminated Plates

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Abstract

SHEAR deformations are considered in laminated plate theory in terms of correction factors k_{ij} (or k'_{ij}) and modified shear stiffnesses $K_{ij}=k_{ij}A_{ij}=k'_{ij}A'_{ij}$; $(i,j=4,5)$, where A_{ij} (or A'_{ij}) are obtained from assumed "constant strain (or stress)" fields.[1] This work reports the correction factors, obtained by matching cutoff frequencies for propagation of thickness shear waves (ratio of wavelength λ to plate thickness H approaching infinity) predicted by plate and elasticity theories.

Contents

For an exact elasticity solution, the stiffness matrix relating tractions to displacements on the surfaces of each layer is expressed as a function of shear moduli and frequency.[2] Cutoff frequencies $\omega_{1,2}$ for the first two modes dominated by strains γ_{xz} and γ_{yz}, respectively, are calculated such that determinant of the global stiffness matrix goes to zero. k'_{ii} are then evaluated as K_{ii}/A'_{ii}, where

$$2K_{55,44}=I'(\omega_1^2+\omega_2^2)\pm[I'^2(\omega_1^2-\omega_2^2)^2-4K^*_{45}K^*_{54}]^{1/2} \qquad (1)$$

where I' is the modified rotary inertia term. K^*_{45},K^*_{54} are obtained from the shear force-strain relation $Q_{i\alpha}=\Sigma K^*_{ij}\psi_{j\alpha}$; $(i=5,4;\ \alpha=1,2)$, where $Q_{i\alpha}$ are sums of stresses τ_{xz}, τ_{yz} for each mode α and $\psi_{j\alpha}=Q_{j\alpha}/(I'\omega_\alpha^2)$ are average shear strains estimated using plate equations. Values of the cross terms are found to be close ($K_{45}=K_{54}$, from reciprocity relation) and an

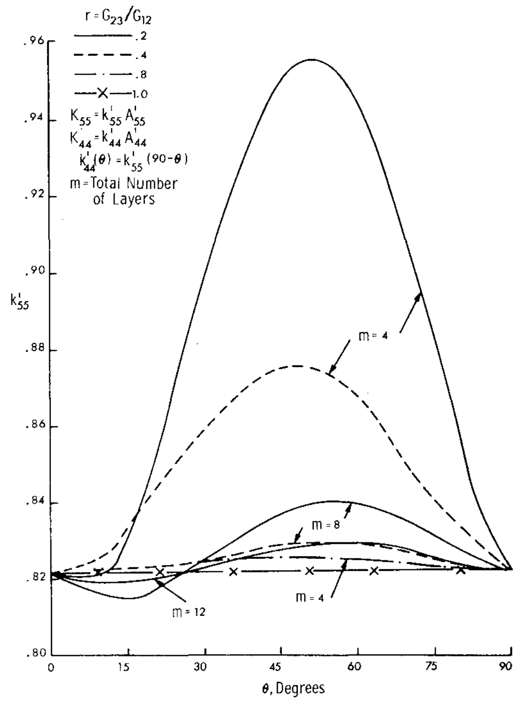

Fig. 2 Correction factors k'_{ii} for angle-ply $[[\pm\theta]_{m/4}]_s$ laminates.

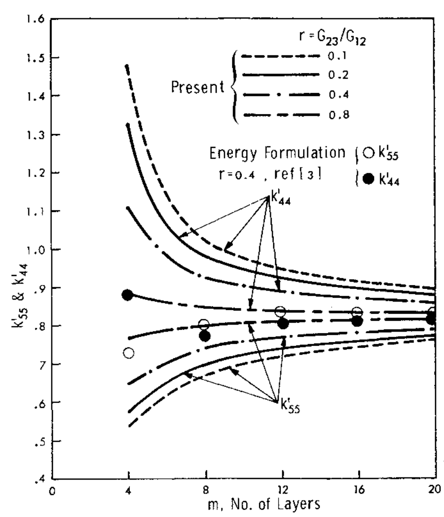

Fig. 1 Correction factors for cross-ply $[[0/90]_{m/4}]_s$ laminates.

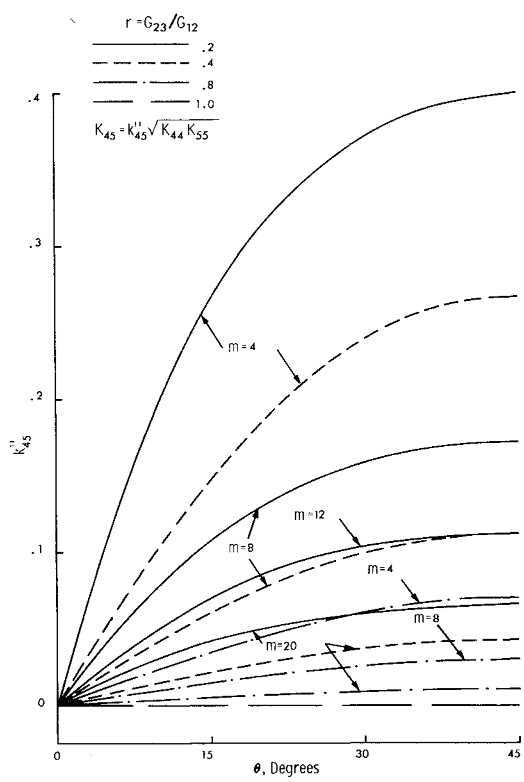

Fig. 3 Correction factor k''_{45} for angle-ply $[[\pm\theta]_{m/4}]_s$ laminates.

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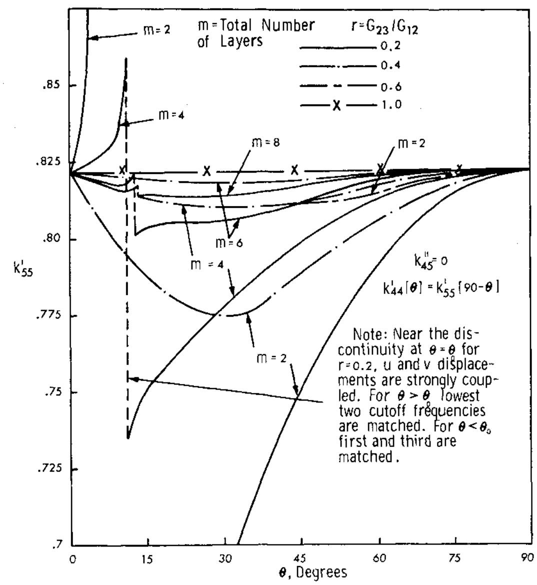

Fig. 4 Correction factors for antisymmetric angle-ply $[\pm\theta]_{m/2}$ laminates.

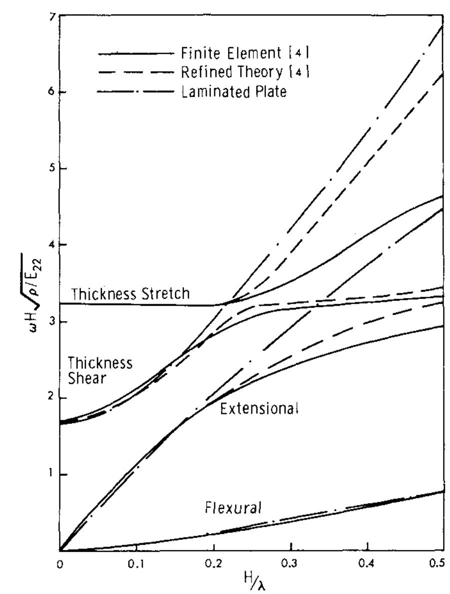

Fig. 5 Frequency spectra for a cross-ply $[0/90]_2$ laminate.

average estimate is $K_{45}^2 = K_{45}^* K_{54}^*$. When $K_{45} \neq 0$, but $A'_{45} = 0$, a nondimensional measure of coupling is $k''_{45} = K_{45}/(K_{44}K_{55})^{1/2}$.

Results for some laminates are given in Figs. 1-4. G_{23} and G_{12} $(=G_{13})$ are transverse and axial shear moduli of each layer. Detailed discussions of the discontinuity in Fig. 4 and

Table 1 Results for laminates and sandwich plate (for prescribed ratios ρ_f/ρ_c of face and core densities) with properties listed in Ref. 3

	k'_{45}		k'_{44}		k''_{45}
	Present	Ref. 3	Present	Ref. 3	Present
Laminate					
±30 deg	0.775	0.826	0.806	1.280	0
$(\pm 30\ \text{deg})_s$	0.859	0.647	0.868	1.120	0.24
[0/90, $N=9$]	0.782	0.844	0.864	0.749	0
Sandwich					
$\rho_f/\rho_c = 1.0$	0.685	0.833	0.714	0.872	0
$\rho_f/\rho_c = 2.0$	0.738	0.833	0.767	0.872	0
$\rho_f/\rho_c = 10.0$	0.808	0.833	0.834	0.872	0

results for other types of laminates can be found in Ref. 2. For obvious reasons, as the number of layers is increased, laminates with a periodic stacking of 0,90 and/or $\pm\theta$ layers tend to behave as a homogeneous plate, i.e, $k'_{ii} \rightarrow \pi^2/12$ and $k''_{45} \rightarrow 0$. Correction factors k_{ii}, however, approach the limiting value $\pi^2 A'_{ii}/12A_{ii}$, which clarifies the contradiction reported in Ref. 2. However, frequency matching applied to laminates considered in Ref. 3 via static formulation yields some differences (Fig. 1 and Table 1) much higher than those observed in a homogeneous plate. In absence of any study on the effects of these differences, it is reasonable to use correction factors from dynamic or static formulation to corresponding problems. Application of present results to a problem considered in Ref. 4 is shown in Fig. 5, which shows that laminate theory with proper correction factors (values used are $k'_{ii} = 0.81$ for $r = 0.4$, same as $[\pm 45]_2$ in Fig. 4) yields accurate results for $\lambda/H \geq 5$. Note that $k_{ii} = 0.66$, close to 0.69 used in refined theory.[4] Other applications are reported in Refs. 2 and 5.

Acknowledgment

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